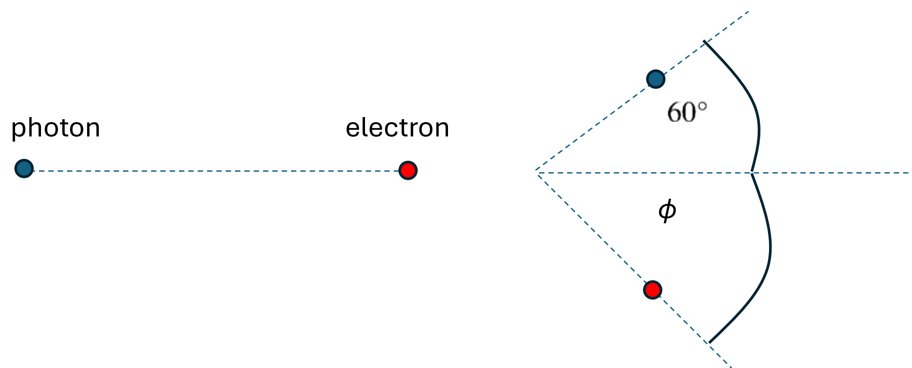
Teacher notes
Topic E

A problem on the Compton effect

A photon of energy $m_e c^2$ (i.e. equal to the rest energy of an electron) is incident on an electron at rest. The photon scatters at an angle of $60°$ relative to its original direction.



Calculate

(i) the incident photon wavelength and momentum,
(ii) the wavelength of the scattered photon,
(iii) the kinetic energy of the electron,
(iv) the momentum of the electron,
(v) the angle at which the electron moves off.

Answers

(i) The photon wavelength and momentum are found from $\lambda = \frac{hc}{E}$; $p = \frac{h}{\lambda}\left(= \frac{E}{c}\right)$.

Hence $\lambda = \frac{hc}{E} = \frac{hc}{m_e c^2} = \frac{h}{m_e c}$. Similarly, $p = \frac{E}{c} = \frac{m_e c^2}{c} = m_e c$.

(ii) $\lambda' - \lambda = \frac{h}{m_e c}(1 - \cos\theta) = \frac{h}{m_e c} \times \left(1 - \frac{1}{2}\right) = \frac{h}{2m_e c}$. Hence, $\lambda' = \frac{h}{2m_e c} + \frac{h}{m_e c} = \frac{3h}{2m_e c}$. (This means that the scattered photon has energy $\frac{hc}{\lambda'} = \frac{hc}{\frac{3h}{2m_e c}} = \frac{2}{3}m_e c^2$ and momentum

$\frac{E'}{c} = \frac{\frac{2}{3}m_e c^2}{c} = \frac{2}{3}m_e c$ or if you prefer $\frac{h}{\lambda'} = \frac{h}{\frac{3h}{2m_e c}} = \frac{2}{3}m_e c$.)

(iii) The energy transferred is $\Delta E = E - E' = m_e c^2 - \frac{2}{3}m_e c^2 = \frac{m_e c^2}{3}$.

(iv) The mistake would be to write $\frac{m_e c^2}{3} = \frac{p^2}{2m_e}$ to get the wrong answer of $p = \sqrt{\frac{2}{3}}m_e c$.

This is wrong because the equation $E_K = \frac{p^2}{2m_e}$ is valid for non-relativistic electrons and the electron here is relativistic. We must apply conservation of momentum to the collision of a photon with the electron.

$$m_e c = \frac{2}{3}m_e c\cos 60 + p\cos\varphi \qquad\qquad p\cos\varphi = \frac{2}{3}m_e c$$

i.e.

$$0 = \frac{2}{3}m_e c\sin 60° - p\sin\varphi \qquad\qquad p\sin\varphi = \frac{\sqrt{3}}{3}m_e c$$

This gives, squaring and adding,

$$p = \sqrt{\left(\frac{4}{9} + \frac{3}{9}\right)}m_e c = \frac{\sqrt{7}}{3}m_e c$$

(We can confirm with the relativistic formula $E^2 = (m_e c^2)^2 + p^2c^2$. Then $E' = m_e c^2 + \frac{m_e c^2}{3} = \frac{4m_e c^2}{3}$ so that $\left(\frac{4m_e c^2}{3}\right)^2 = (m_e c^2)^2 + p^2c^2$ which solves for the momentum as $p^2c^2 = (m_e c^2)^2 \times \left(\frac{16}{9} - 1\right) = (m_e c^2)^2 \times \frac{7}{9}$. Hence $p = \frac{\sqrt{7}}{3}m_e c$ as before.)

(v) From

$$p\cos\varphi = \frac{2}{3}m_e c$$

$$p\sin\varphi = \frac{\sqrt{3}}{3}m_e c$$

we get, by dividing side by side,

$$\tan\varphi = \frac{\frac{\sqrt{3}}{3}}{\frac{2}{3}} = \frac{\sqrt{3}}{2}$$ i.e. $\varphi \approx 40.9°$.